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Question Paper Code : 77196

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Computer Science and Engineering

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulation 2013)

Time : Three hours — Maximum : 100 marks

Use of statistical tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Test whether $f(x)=\begin{cases}|x|, -1\le x\le 1\\ 0, \ otherwise\end{cases}$

   can be the probability density function of a continuous random variable?

2. What are the limitations of Poisson distribution?

3. The joint pdf of a two-dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases}kxe^{-y}; 0\le x\le 2,\ y>0\\ 0, \ otherwise\end{cases}$. Find the value of '$k$'.

4. In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible: Variance of $X = 9$; Regression equations are $8X-10Y+66=0$ and $40X-18Y=214$. What are the mean values of X and Y?

5. Is Poisson process stationary? Justify.

6. What is a random process? When do you say a random process is a random variable?

7. What do the letters in the symbolic representation $(a/b/c)\,(d/e)$ of a queueing model represent?

8. What do you mean by balking and reneging?

9. State Jackson's theorem for an open network.

10. What do you mean by E_k in the $M/E_k/1$ queueing model?

PART B — (5 × 16 = 80 marks)

11. (a) (i) The CDF of the random variable of X is given by

$$F_X(x)=\begin{cases}0; x<0\\ x+\frac{1}{2}; 0\le x\le\frac{1}{2}.\\ 1 \quad ; x>\frac{1}{2}\end{cases}$$

Draw the graph of the CDF. Compute $P(X>1/4)$, $P\left(\frac{1}{3}<X\le\frac{1}{2}\right)$. (8)

(ii) Find the moment generating function of a geometrically distributed random variable and hence find the mean and variance. (8)

Or

(b) (i) Messages arrive at a switch board in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

(1) exactly two messages arrive within one hour

(2) no message arrives within one hour

(3) at least three messages arrive within one hour. (8)

(ii) The peak temperature T, as measured in degrees Fahrenheit, on a particular day is the Gaussian (85 ,10) random variable. What is $P(T>100)$, $P(T<60)$ and $P(70\le T\le 100)$? (8)

12. (a) (i) Find the equation of the regression line Y on X from the following data : (10)

X	3	5	6	8	9	11
Y	2	3	4	6	5	8

(ii) Assume that the random variables X and Y have the joint PDF $f(x,y)=\frac{1}{2}x^3y; 0\le x\le 2, 0\le y\le 1$. Determine if X and Y are independent. (6)

Or

(b) The joint PDF of the random variables X and Y is defined as

$$f(x,y)=\begin{cases}25e^{-5y}; 0<x<0.2, y>0\\ 0, \textit{ otherwise}\end{cases}$$

(i) Find the marginal PDFs and X and Y

(ii) What is the covariance of X and Y? (16)

13. (a) (i) A random process $X(t)$ is defined by $X(t) = A\cos t + B\sin t$, $-\infty < t < \infty$ where A and B are independent random variables each of which has a value –2 with probability 1/3 and a value 1 with probability 2/3. Show that $X(t)$ is a wide-sense stationary process. (8)

(ii) An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the limiting probability of the signals generated by the testing system are highly distorted. (8)

Or

(b) (i) State and prove Chapman Kolmogorov equation. (8)

(ii) A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Prove that $\{X_n; n = 0, 1, 2....\}$ is a Markov chain. Also, find its transition probability matrix and draw its transition graph. (8)

14. (a) (i) Explain Markovian Birth - Death process and obtain the expressions for steady state probabilities. (8)

(ii) A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in Poisson fashion at the rate of 10 per hour. What is the probability that a customers has to wait for service? (8)

Or

(b) (i) A small mail-order business has one telephone line and a facility for call waiting for two additional customers. Orders arrive at the rate of one per minute and each order requires 2 minutes and 30 seconds to take down the particulars. What is the expected number of calls waiting in the queue? What is the mean waiting time in the queue? (8)

(ii) An airport has a single runway. Airplanes have been found to arrive at the rate of 15 per hour. It is estimated that each landing takes 3 minutes. Assuming a Poisson process for arrivals and an exponential distribution for landing times. Find the expected number of airplanes waiting to land, expected waiting time. What is the probability that the waiting will be more than 5minutes? (8)

15. (a) Derive Pollaczek — Khintchine formula for the average number of customers in the M/G/1 queueing system. (16)

Or

(b) (i) Write a short note on open queueing network. (8)

(ii) Patients arrive at a clinic in a Poisson fashion at the rate of 3 per hour. Each arriving patients has to pass through two sections. The assistant in the first section take 15 minutes per patient and the doctor in the second section takes nearly 6 minutes per patient. If the service times in the two sections are approximately exponential, find the probability that there are 3 patients in the first sections and 2 patients in the second section. Find also average number of patients in the clinic and the average waiting time of a patient. (8)