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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fourth Semester

Biomedical Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering/Robotics and Automation Engineering)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Write the formula for moment generating function of binomial distribution.

2. Suppose that the duration X in minutes of long distance calls from your home, follows exponential law with p.d.f. $f(x)=\begin{cases} e^{\frac{-x}{5}}, x>0 \\ 0, \text{otherwise} \end{cases}$ what is $P(X > 5)$?

3. Find the value of k, if $f(x, y) = k(1 - x)(1 - y)$ in $0 < x, y < 1$ and $f(x, y) = 0$, otherwise, is to be the joint density function.

4. The regression equations are $3x + 2y = 26$ and $6x + y = 31$. Find the means of X and Y.

5. What do you mean by wide sense stationary process ?

6. State the postulates of a Poisson process.

7. Prove that $R(\tau)$ is maximum at $\tau = 0$.

8. Find the variance of the stationary process {x(t)} whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

9. State unit impulse response of a system. Why is it called so ?

10. Define band limited white noise.

PART – B (5×16=80 Marks)

11. a) i) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown. (8)

ii) If the probability that a target is destroyed on any one shot is 0.5. What is the probability that it would be destroyed on 6^{th} attempt ? (8)

(OR)

b) i) Find the mean and variance of Gamma distribution. (8)

ii) In a test on 2000 electric bulbs, it was found that the life of a Philips bulbs was normally distributed with an average of 2400 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2150 hours, (ii) less than 1950 hours. (8)

12. a) i) The waiting times X and Y of two customers entering a bank at different times are assumed to be independent random variables with p.d.f.s

$$f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, \text{otherwise} \end{cases} \quad \text{and} \quad f(x) = \begin{cases} e^{-y}, y > 0 \\ 0, \text{otherwise} \end{cases}$$

Find the joint p.d.f. of $U = X + Y$, $V = \dfrac{X}{X+Y}$. (8)

ii) The two dimensional random variable (X, Y) has the joint density function $f(x, y) = (x + 7y)/27$, $x = 0, 1, 2$, $y = 0, 1, 2$. Obtain $f(y \mid x)$ and $f(x \mid y = 1)$. (8)

(OR)

b) The joint pdf of the random variable (X, Y) iff $(x, y) = (x + y)$, $0 \le x \le 1$, $0 \le y \le 1$, find cov (X, Y). (16)

13. a) i) Examine whether the random process $\{X(t)\} = A\cos(wt + \theta)$ is a wide sense stationary if A and w are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (8)

ii) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train. But he drives one day, then the next day is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a train on the fourth day and the probability that he drives to work on the fifth day. (8)

(OR)

b) i) A Hard Disk fails in a computer system and it follows a Poisson distribution with mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If we have 5 extra hard disks and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in next 10 weeks. (8)

ii) Define random telegraph signal process and prove that it is wide-sense stationary. (8)

14. a) i) Find the power spectral density of the random process whose auto correlation function $R(\tau) = \begin{cases} 1 - |\tau|, & |\tau| \le 1 \\ 0, & \text{otherwise} \end{cases}$. (8)

ii) If the power spectral density of a WSS process is given by

$$S(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|, & |\omega| \le a \\ |\omega|, & |\omega| > a \end{cases}$$, find the autocorrelation function of the process. (8)

(OR)

b) i) If the cross power spectral density of X(t) and Y(t) is

$$S(\omega) = \begin{cases} \left(a + \frac{ib\omega}{\alpha}\right), & |\omega| \le \alpha, \alpha > 0 \\ 0, & \text{otherwise} \end{cases}$$, where a and b are constants. Find the cross correlation function. (8)

ii) Let X(t) and Y(t) be both zero-mean and WSS random processes consider the random process Z(t) defined by Z(t) = X(t) + Y(t). Find autocorrelation function and the power spectrum of Z(t) if X(t) and Y(t) are jointly WSS. (8)

15. a) If $\{X(t)\}$ is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)\, X(t-u)\, du$, prove that : **(16)**

i) $R_{XY}(T) = R_{XX}(T) * h(-T)$, where * denotes convolution

ii) $R_{YY}(T) = R_{XY}(T) * h(T)$, where * denotes convolution

iii) $S_{XY}(\omega) = S_{XX}(\omega) H^*(\omega)$, $H^*(\omega)$ is the complex conjugate of $H(\omega)$

iv) $S_{XY}(\omega) = S_{XX}(\omega) \mid H(\omega) \mid^2$.

(OR)

b) i) If X(t) is the input voltage to a circuit and Y(t) is the output voltage, $\{X(t)\}$ is a stationary random process with $\mu_x = 0$, and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find μ_y, $S_{yy}(\omega)$ and $R_{yy}(T)$, if the power transfer function is $H(\omega) = \dfrac{R}{R + iL\omega}$. **(8)**

ii) A system has an impulse response $h(t) = e^{-\beta t}\, U(t)$, find the power spectral density of the output Y(t) corresponding to the input X(t). **(8)**