

- (ii) Two jointly stationary random processes $X(t)$ and $Y(t)$ have the cross power spectral density given by $S_{xy}(w) = \frac{1}{-w^2 + j4w + 4}$. Find the corresponding cross correlation function. (8)

15. (a) (i) A random process $\{X(t)\}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. If the autocorrelation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $Y(t)$. (10)
- (ii) A linear system has a transfer function given by $H(w) = \frac{w}{w^2 + 15w + 50}$. Determine the power spectral density of the output when the input function is white noise that has a mean square value of $1.2V^2/Hz$. (6)

Or

- (b) $X(t)$ is a Wide-Sense stationary process that is the input to a linear system with the transfer function $H(w) = \frac{1}{a + jw}$ where $a > 0$. If $X(t)$ is a zero-mean white noise with power spectral density $\frac{N_0}{2}$, determine the following
- (i) The impulse response $h(t)$ of the system
- (ii) The cross-power spectral density $S_{XY}(w)$ of the input process and the output process $Y(t)$
- (iii) The cross correlation function $R_{YX}(\tau)$ of $Y(t)$ and $X(t)$
- (iv) The power spectral density $S_{YY}(w)$ of the output process. (4 × 4 = 16)

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B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering/
Electronics and Telecommunication Engineering)

(Regulation 2017)

Time : Three hours

Maximum : 100 marks

(Use of statistical table is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

- Show that for any events A and B in S , $P(B) = P(B/A)P(A) + P(B/\bar{A})P(\bar{A})$.
- Find the second moment about the origin of the Geometric distribution with parameter p .
- The joint pdf of a bivariate random variable (X, Y) is given by $f_{xy}(x, y) = \begin{cases} k, & 0 < y \leq x < 1 \\ 0, & \text{otherwise} \end{cases}$ where k is a constant. Determine the value of k .
- Define covariance and coefficient of correlation between two random variables x and y .
- Define Markov process.
- Examine whether the Poisson process $\{X(t)\}$, given by the probability law $P\{X(t) = r\} = \frac{e^{-\lambda t} (\lambda t)^r}{r!}$, $r = 0, 1, 2, \dots$ is covariance stationary.
- State any two properties of auto correlation function.
- Show that the power spectrum of a (real) random process $\{X(E)\}$ is real.

9. State fundamental theorem on the power spectrum of the output of a linear system.

10. Find the system transfer function, if a linear time invariant system has an

$$\text{impulse function } H(t) = \begin{cases} \frac{1}{2c}, & |t| \leq c \\ 0, & \text{otherwise} \end{cases}$$

PART B — (5 × 16 = 80 marks)

11. (a) (i) State and prove Baye's theorem. (8)
 (ii) Derive the moment generating function of normal distribution. (8)

Or

- (b) (i) Derive the moment generation function of Poisson distribution and hence find its first three central moments. (8)
 (ii) Out of 800 families with 4 children each, how many families would be expected to have (8)
 (1) 2 boys and 2 girls
 (2) atleast one boy
 (3) atleast two girls
 (4) children of both sexes.

Assume equal probabilities for boys and girls.

12. (a) (i) Determine if random variables X and Y are independent when their joint PDF is given by

$$f_{xy}(x, y) = \begin{cases} \frac{1}{2} e^{-(x+y)}, & 0 \leq x \leq y, 0 \leq y < \infty \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

- (ii) Assume that the random variable S is the sum of 48 independent experimental values of the random variable X whose PDF is given by

$$f_x(x) = \begin{cases} \frac{1}{3}, & 1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that S lies in the range (108, 126). (8)

Or

- (b) Two independent variables X and Y are define by $f(x) = 4ax$, $0 \leq x \leq r$, $f(y) = 4by$, $0 \leq y \leq s$ show that $r(U, V) = \frac{b-a}{b+a}$ where $U = X + Y$ and $V = X - Y$. (16)

13. (a) (i) The random process $\{X(t)\}$ is defined as $X(t) = 2e^{-At} \sin(\omega t + B)u(t)$ where $u(t)$ is the unit step function and the random variables A and B are independent. A is uniformly distributed in $(0, 2)$ and B is uniformly distributed in $(-\pi, \pi)$. Verify whether the process is wide sense stationary. (10)

- (ii) Prove that the inter arrival time of a poisson process with parameter λ has an exponential distribution with mean $\frac{1}{\lambda}$. (6)

Or

- (b) (i) Find the nature of the states of the Markov Chain with three states 0, 1, 2 and with one step transition probability matrix,

$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \quad (8)$$

- (ii) If $\{X_1(t)\}$ and $\{X_2(t)\}$ represent two independent poisson processes with parameters $\lambda_1 t$ and $\lambda_2 t$ respectively, then prove that $P\{X_1(t) = X / X_1(t) + X_2(t) = n\}$ is Binomial with parameters n and p where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. (8)

14. (a) (i) A stationary random process $\{X(t)\}$ has the power spectral density $S_{xx}(\omega) = \frac{24}{\omega^2 + 16}$. Find the mean-square value of the process by Brute-Force method. (10)

- (ii) Prove that autocorrelation function of the random process with the power spectral density given by $S_{xx}(\omega) = \begin{cases} s_0, & |w| < w_0 \\ 0, & \text{otherwise} \end{cases}$ is

$$\frac{s_0}{\pi \tau} \sin \omega_0 \tau \quad (6)$$

Or

- (b) (i) Two random processes $X(t)$ and $Y(t)$ are defined as follows.

$$X(t) = A \cos(\omega t + \theta)$$

$$Y(t) = B \sin(\omega t + \theta)$$

where A, B and w are constant and θ is a random variable that is uniformly distributed between 0 and 2π . Find the cross correlation function of $X(t)$ and $Y(t)$ and show that $X(t)$ and $Y(t)$ are jointly WSS. (8)